Test 2 / Numerical Mathematics 1 / May 30th 2023, University of Groningen

Instructions

- **Write the answers to Exercises 1 and 2 on separate sheets.**
- **Use a ball-point pen (blue or black ink) to write your answers.**
- You have 2 hours to complete the test. When applicable, people with special facilities have 2h20 minutes in total.
- The exam is "closed book", meaning that you can only make use of the material given to you.
- A simple calculator is allowed.
- The grade will be computed as the number of obtained points, plus 1.
- Not complying with the aforementioned rules will lead to zero points.

Exercise 1 (6 points)

Consider the problem: Find x^* such that $g(x^*) = 0$ solved via the iterations

$$x^{(n+1)} = p(x^{(n)}) := x^{(n)} - g(x^{(n)})/g'(x^{(n)}) \tag{1}$$

(a) [2] Show that if $g(x) = (x - x^*)^q h(x)$, $q > 0$, $h(x^*) \neq 0$, then $p'(x^*) = 1 - 1/q$.

(b) [1.5] Using iterations (1) for finding the root of $g(x) = x^{2.5}$ leads to the following iterands

n	0	1	2	3	4	5
$x^{(n)}$	20	12	7.2	4.32	2.592	1.5552

Compute the convergence order of these iterations and explain the obtained value using the theory studied in the course.

(c) [1] Propose a new iteration function $w(x)$ (depending on the specific $g(x)$ given in the first question), by modifying $p(x)$ in order to obtain a higher order of convergence. Justify according to the theory seen in class and tutorials.

(d) [1.5] Using the modified iteration function $w(x)$, perform a few iterations to approximate the root of $x^{2.5}$, with $x^{(0)} = 20$. Explain the difference in the convergence order with respect to the iterations (1).

Exercise 2 (3 points)

Consider solving the linear system of equations $By = a$, with $B \in \mathbb{R}^{n \times n}$, using the following iterative procedure

$$y_k = y_{k-1} + \gamma \left(B y_{k-1} - a \right), \quad k \geq 1, \tag{2}$$

with

$$B = \begin{bmatrix} 2 & -1 \\ 1 & -3 \end{bmatrix}, a = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

(e) [2] Is it possible to choose a constant value of γ such that the iterations (2) converge? Justify your answer just by recalling the proofs studied in the course.

(f) [1] In case you answered "yes" in the previous question, give a value of γ that makes the iterations converge. If you answered "no", re-write the linear system (such that the solution is the same) so you can pick a value of γ for making the solutions converge.